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INVESTIGATION OF WAVE PROCESSES IN A THERMOELASTIC MICROPOLAR SOLID BODY BY THE METHOD OF THE THEORY OF CHARACTERISTICS

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Consideration is given to an elastic medium with a finite time of relaxation of the heat flux; the equations of motion of the medium are written in components of the tensors of force and couple stresses. Using the general theory of characteristics the explicit formulas for determination of the velocities of propagation of the discontinuity surfaces are obtained and the equations of the characteristic surfaces are derived.

The issue of thermoelastic stresses in a micropolar isotropic medium has been considered by a number of authors [1–4]. The problem of the existence of nonstationary processes in such media is investigated in [5, 6]. Below we analyze the propagation of discontinuity surfaces in a two-dimensional micropolar medium with a finite time of relaxation of the heat flux in the context of the theory of characteristics of partial differential equations. The expediency of such an approach is explained by the fact that the applications of this method in specific divisions of the mechanics of continuous media are associated with overcoming significant difficulties; therefore, its realization is of both theoretical and practical value.

The stressed-strained state of an elastic isotropic micropolar body is described by the tensors of force and couple stresses of the following form [7]:

$$\sigma_{ki} = \lambda \delta_{ki} e_{nn} + (\mu + \alpha)\, e_{ki} + (\mu - \alpha)\, e_{ik} - \nu\theta\,, \tag{1}$$

$$\mu_{ki} = \beta \delta_{ki}\, \varphi_{m,m} + (\gamma + \varepsilon)\, \varphi_{k,i} + (\gamma - \varepsilon)\, \varphi_{i,k}\,, \tag{2}$$

where $e_{ki} = u_{k,i} + \varepsilon_{kim}\varphi_m$ is the microstrain tensor, $\mathbf{u} = (u_1, u_2, u_3)$ is the displacement vector, $\varphi = (\varphi_1, \varphi_2, \varphi_3)$ is the microrotation vector, ε_{kim} is the Levi–Civita pseudotensor, δ_{ki} is the Kronecker tensor, $e_{nn} = e_{11} + e_{22} + e_{33}$, and $\varphi_{m,m} = \varphi_{1,1} + \varphi_{2,2} + \varphi_{3,3}$, $i, k, m, n = \overline{1, 3}$. Let us substitute (1)–(2) into the equations of motion [7]

$$\sigma_{ki,k} + X_i = \rho \ddot{u}_i\,, \tag{3}$$

$$\mu_{ki,k} + \varepsilon_{imn}\sigma_{mn} + Y_i = j\rho\ddot{\varphi}_i\,. \tag{4}$$

Here X_i and Y_i are the mass forces and the body couples, j is the measure of rotary inertia, and $i, k, m, n = \overline{1, 3}$. We have [7]

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$$(\mu+\alpha)\,\Delta u_i+(\lambda+\mu-\alpha)\,u_{k,ik}+2\alpha\varepsilon_{ikl}\,\varphi_{l,k}+X_i=\rho\ddot{u}_i+\nu\theta_{,i}\,, \tag{5}$$

$$(\gamma+\varepsilon)\,\Delta\varphi_i+(\beta+\gamma-\varepsilon)\,\varphi_{k,ik}+4\alpha\varepsilon_{ikl}u_{l,k}-4\alpha\varphi_i+Y_i=j\rho\ddot{\varphi}_i\,. \tag{6}$$

In order to write the last system in components of the tensors of force and couple stresses we differentiate Eqs. (5) and (6) with respect to x_j. We obtain

$$(\mu+\alpha)\,\Delta u_{i,j}+(\lambda+\mu-\alpha)\,u_{k,kij}+2\alpha\varepsilon_{ikl}\,\varphi_{l,kj}+X_{i,j}=\rho\ddot{u}_{i,j}+\nu\theta_{,ij}\,, \tag{7}$$

$$(\gamma+\varepsilon)\,\Delta\varphi_{i,j}+(\beta+\gamma-\varepsilon)\,\varphi_{k,kij}+4\alpha\varepsilon_{ikl}u_{l,kj}-4\alpha\varphi_{i,j}+Y_{i,j}=j\rho\ddot{\varphi}_{i,j}\,. \tag{8}$$

Under conditions of plane strain, the micropolar elastic body is characterized by the following matrices of the force- and couple-stress tensors [7]:

$$\sigma=\begin{Vmatrix}\sigma_{11} & \sigma_{12} & 0\\ \sigma_{21} & \sigma_{22} & 0\\ 0 & 0 & \sigma_{33}\end{Vmatrix}\,,\quad \mu=\begin{Vmatrix}0 & 0 & \mu_{13}\\ 0 & 0 & \mu_{23}\\ \mu_{31} & \mu_{32} & 0\end{Vmatrix}\,.$$

In this case, the components [7] σ_{33}, μ_{31}, and μ_{32} are calculated from the formulas

$$\sigma_{33}=\frac{(\sigma_{11}+\sigma_{22})\,\lambda}{2\,(\lambda+\mu)}+\frac{\mu\nu}{\lambda+\mu}\,\theta\,,\quad \mu_{31}=\frac{\gamma-\varepsilon}{\gamma+\varepsilon}\,\mu_{13}\,,\quad \mu_{32}=\frac{\gamma-\varepsilon}{\gamma+\varepsilon}\,\mu_{23}\,.$$

Therefore, the system of equations of motion will include six equations relative to σ_{ij} and μ_{i3}, i, j = 1, 2. From (1), (2), and (4) we have

$$\begin{gathered}
u_{i,i}=\frac{1}{2\mu}\left(\sigma_{ii}-\frac{\lambda}{2\,(\lambda+\mu)}\,(\sigma_{11}+\sigma_{22})\right)+\frac{\nu\theta}{2\,(\lambda+\mu)}\,,\\
u_{1,2}=\frac{1}{4\mu}\,(\sigma_{12}+\sigma_{21})+\frac{1}{4\alpha}\,(\sigma_{12}-\sigma_{21})+\varphi_3\,,\ \ \varphi_{1,3}=\frac{\mu_{13}}{\gamma+\varepsilon}\,,\\
u_{2,1}=\frac{1}{4\mu}\,(\sigma_{12}+\sigma_{21})-\frac{1}{4\alpha}\,(\sigma_{12}-\sigma_{21})-\varphi_3\,,\ \ \varphi_{2,3}=\frac{\mu_{23}}{\gamma+\varepsilon}\,,\\
\ddot{\varphi}_3=\frac{1}{j\rho}\,(\mu_{13,1}+\mu_{23,2}+\sigma_{12}-\sigma_{21}+Y_3)\,.
\end{gathered} \tag{9}$$

Upon simple transformations, system (7) and (8) will take the form

$$(\lambda+2\mu)\,\Delta\,(\sigma_{11}+\sigma_{22})+2\nu\,(\mu\Delta\theta-\rho\ddot{\theta})+X_{1,1}+X_{2,2}=\rho\,(\ddot{\sigma}_{11}+\ddot{\sigma}_{22})\,,$$

$$\frac{\mu+\alpha}{2\mu}\,\Delta\,(\sigma_{22}-\sigma_{11})+\frac{\lambda+\mu-\alpha}{2\,(\lambda+\mu)}\,(\sigma_{22,22}-\sigma_{22,11}+\sigma_{11,22}-\sigma_{11,11})-$$

$$-\frac{\alpha\nu}{\lambda+\mu}\,(\theta_{,11}-\theta_{,22})+\frac{4\alpha}{\gamma+\varepsilon}\,\mu_{23,1}+X_{2,2}-X_{1,1}=\frac{\rho}{2\mu}\,(\ddot{\sigma}_{22}-\ddot{\sigma}_{11})\,,$$

$$\frac{\mu+\alpha}{2\mu}\,\Delta\,(\sigma_{12}+\sigma_{21})+\frac{\lambda+\mu-\alpha}{\lambda+\mu}\,(\sigma_{11,12}+\sigma_{22,12})-\frac{2\alpha\nu}{\lambda+\mu}\,\theta_{,12}+$$

$$+\frac{2\alpha}{\gamma+\varepsilon}\,(\mu_{13,1}-\mu_{23,2})+X_{1,2}+X_{2,1}=\frac{\rho}{2\mu}\,(\ddot{\sigma}_{12}+\ddot{\sigma}_{21})\;, \tag{10}$$

$$\frac{\mu+\alpha}{2\alpha}\,\Delta\,(\sigma_{12}-\sigma_{21})+\frac{2\,(\mu+\alpha)}{\gamma+\varepsilon}\,(\mu_{13,1}+\mu_{23,2})+X_{2,1}-X_{1,2}=$$

$$=\frac{\rho}{2\alpha}\,(\ddot{\sigma}_{12}-\ddot{\sigma}_{21})+\frac{2}{j}\,(\mu_{13,1}+\mu_{23,2}+\sigma_{12}-\sigma_{21}+Y_3)\;,$$

$$\Delta\mu_{13}+2\,(\sigma_{12,1}-\sigma_{21,1})+Y_{3,1}=\frac{j\rho}{\gamma+\varepsilon}\,\ddot{\mu}_{13}\;,\quad \Delta\mu_{23}+2\,(\sigma_{12,2}-\sigma_{21,2})+Y_{3,2}=\frac{j\rho}{\gamma+\varepsilon}\,\ddot{\mu}_{23}\;.$$

Here

$$\Delta=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\;.$$

For this system to be closed we add the hyperbolic law of thermoelasticity to it, which for a two-dimensional problem has the form [4, 8]

$$k\Delta\theta-c_v\,(\dot{\theta}+\tau\ddot{\theta})=\nu\theta_0\,(\tau\,(\ddot{e}_{11}+\ddot{e}_{22})+\dot{e}_{11}+\dot{e}_{22})\;.$$

Applying relations (9) to the last equation, we obtain

$$k\Delta\theta-\left(c_v+\frac{v^2\theta_0}{\lambda+\mu}\right)(\dot{\theta}+\tau\ddot{\theta})=\frac{\nu\theta_0}{2\,(\lambda+\mu)}\,(\tau\,(\ddot{\sigma}_{11}+\ddot{\sigma}_{22})+\dot{\sigma}_{11}+\dot{\sigma}_{22})\;. \tag{11}$$

We specify the initial data for system (10)–(11) on the surface $Z(t,\,x_1,\,x_2)$ = const and pass to new variables according to the formulas [9, 10]

$$\frac{\partial y_j\,(t,X)}{\partial x_k}=\sum_{l=0}^{2}\frac{\partial y_j}{\partial Z_l}\frac{\partial Z_l}{\partial x_k}\,,$$

$$\frac{\partial^2 y_j}{\partial x_k\partial x_n}=\sum_{l,m=0}^{2}\frac{\partial^2 y_j}{\partial Z_l\partial Z_m}\frac{\partial Z_l}{\partial x_k}\frac{\partial Z_m}{\partial x_n}+\sum_{i=0}^{2}\frac{\partial y_j}{\partial Z_l}\frac{\partial^2 Z_l}{\partial x_n\partial x_k}\,, \tag{12}$$

$$Z\equiv Z_0\,,\quad t\equiv x_0\,.$$

We substitute relations (12) into Eqs. (10)–(11) and write those terms that contain the partial derivatives of second order in Z, since only they will be important in what follows [9, 10]. As a result we will have

$$\left((\lambda+\mu)\,g^2-\rho p_0^2\right)\left(\frac{\partial^2\sigma_{11}}{\partial Z^2}+\frac{\partial^2\sigma_{22}}{\partial Z^2}\right)+2\nu\,\frac{\partial^2\theta}{\partial Z^2}\left(\mu g^2-\rho p_0^2\right)+\ldots=0\;,$$

$$\left(\frac{\mu+\alpha}{2\mu}g^2-\frac{\rho}{2\mu}p_0^2\right)\left(\frac{\partial^2\sigma_{22}}{\partial Z^2}-\frac{\partial^2\sigma_{11}}{\partial Z^2}\right)+\frac{\lambda+\mu-\alpha}{2\,(\lambda+\mu)}\left(p_2^2-p_1^2\right)\times$$

$$\times\left(\frac{\partial^2\sigma_{11}}{\partial Z^2}+\frac{\partial^2\sigma_{22}}{\partial Z^2}\right)-\frac{\alpha\nu}{\lambda+\mu}\frac{\partial^2\theta}{\partial Z^2}\left(p_2^2-p_1^2\right)+\ldots=0\,,$$

$$\left(\frac{\mu+\alpha}{2\mu}g^2-\frac{\rho}{2\mu}p_0^2\right)\left(\frac{\partial^2\sigma_{12}}{\partial Z^2}+\frac{\partial^2\sigma_{21}}{\partial Z^2}\right)+\frac{\lambda+\mu-\alpha}{\lambda+\mu}p_1^2p_2^2\times$$

$$\times\left(\frac{\partial^2\sigma_{11}}{\partial Z^2}+\frac{\partial^2\sigma_{22}}{\partial Z^2}\right)-\frac{2\alpha\nu}{\lambda+\mu}\frac{\partial^2\theta}{\partial Z^2}p_1^2p_2^2+\ldots=0\,,$$

$$\left(\frac{\mu+\alpha}{2\alpha}g^2-\frac{\rho}{2\alpha}p_0^2\right)\left(\frac{\partial^2\sigma_{12}}{\partial Z^2}-\frac{\partial^2\sigma_{21}}{\partial Z^2}\right)+\ldots=0\,,$$

$$\frac{\partial^2\mu_{13}}{\partial Z^2}\left(g^2-\frac{j\rho}{\gamma+\varepsilon}p_0^2\right)+\ldots=0\,,\quad \frac{\partial^2\mu_{23}}{\partial Z^2}\left(g^2-\frac{j\rho}{\gamma+\varepsilon}p_0^2\right)+\ldots=0\,,$$

$$\left(kg^2-(a+c_v)\,\tau p_0^2\right)\frac{\partial^2\theta}{\partial Z^2}-b\tau p_0^2\left(\frac{\partial^2\sigma_{11}}{\partial Z^2}+\frac{\partial^2\sigma_{22}}{\partial Z^2}\right)+\ldots=0\,,$$

where

$$p_0=\frac{\partial Z}{\partial t}\,;\quad p_k=\frac{\partial Z}{\partial x_k}\,;\quad g^2=p_1^2+p_2^2\,;\quad a=\frac{\nu^2\theta_0}{\lambda+\mu}\,;\quad b=\frac{\nu\theta_0}{2\,(\lambda+\mu)}\,,\quad k=1,2\,.$$

The equation of the characteristic surface $Z(t, x_1, x_2)$ = const will be obtained from the condition of unsolvability of the last system of equations relative to the derivatives $\frac{\partial^2\sigma_{ij}}{\partial Z^2}$, $\frac{\partial^2\mu_{i3}}{\partial Z^2}$, and $\frac{\partial^2\theta}{\partial Z^2}$, i, j = 1, 2, i.e., from the condition that the determinant composed of the coefficients of these derivatives is equal to zero [9, 10]:

$$\det\|\omega_{kl}\|=0\,, \tag{13}$$

where

$$\omega_{11}=\omega_{12}=(\lambda+\mu)\,g^2-\rho p_0^2\,;\quad \omega_{43}=\omega_{34}=\omega_{33}=-\,\omega_{44}=\frac{\mu+\alpha}{2\mu}g^2-\frac{\rho}{2\mu}p_0^2\,;$$

$$\omega_{21}=-\frac{\mu+\alpha}{2\mu}+\frac{\lambda+\mu-\alpha}{2\,(\lambda+\mu)}(p_2^2-p_1^2)+\frac{\rho}{2\mu}p_0^2\,;\quad \omega_{22}=\frac{\mu+\alpha}{2\mu}+\frac{\lambda+\mu-\alpha}{2\,(\lambda+\mu)}(p_2^2-p_1^2)-\frac{\rho}{2\mu}p_0^2\,;$$

$$\omega_{31}=\omega_{32}=\frac{\lambda+\mu-\alpha}{\lambda+\mu}p_1p_2\,,\quad \omega_{55}=\omega_{66}=g^2-\frac{j\rho}{\gamma+\varepsilon}p_0^2\,;$$

$$\omega_{77}=kg^2-(a+c_v)\,\tau p_0^2\,,\quad \omega_{72}=\omega_{71}=-\,b\tau p_0^2\,,\quad \omega_{17}=2\nu\,(\mu g^2-\rho p_0^2)\,;$$

TABLE 1. Values of the Connectivity Coefficients *a* and *b*

Material	Elastic constants, 10^{10} N/m^2		Thermoelastic constant ν, 10^3 N/(m2·deg)	Connectivity coefficients	
	λ	μ		a, N/(m^2·deg)	b
Silver	8.108	3.378	5905.2	88955	0.0075
Lead	4.006	1.012	3980.6	92527	0.012
Molybdenum	18.880	12.280	4060.0	15500	0.00191

$$\omega_{27} = \frac{\alpha\nu}{\lambda+\mu}\,(p_1^2 - p_2^2)\,, \quad \omega_{37} = -\frac{2\alpha\nu}{\lambda+\mu}\,p_1 p_2\,.$$

All the remaining ω_{kl}, k and $l = \overline{1, 7}$, are equal to zero. We note that the constants a and b determine the conjugation of the thermal field and the strain field; b is of the order of 10^{-2}–10^{-3}, while the constant a is approximately equal to 10^4–10^5 N/(m^2·deg) (Table 1 and [11, 12]). Therefore, the components $\omega_{7i} = -b\tau p_0^2$, i = 1, 2, can be disregarded since b is small and has the order of 10^{-13}–10^{-14} sec in the product with τ (for metals $\tau \sim 10^{-11}$ sec [13]).

From (13), without taking account of ω_{7i}, i = 1, 2, we obtain

$$\left((\mu+\alpha)\,g^2 - \rho p_0^2\right)^3 \left((\gamma+\varepsilon)\,g^2 - j\rho p_0^2\right)^2 \left((\lambda+2\mu+\alpha)\,g^2 - \rho p_0^2\right)\left(kg^2 - \tau\,(c_v + a)\,p_0^2\right) = 0\,.$$

This yields the following velocities of propagation of the discontinuity surfaces $V = -p_0/g$ [3, 9]:

$$V_1 = \sqrt{\frac{\gamma+\varepsilon}{\rho}}\,, \quad V_2 = \sqrt{\frac{\mu+\alpha}{\rho}}\,, \quad V_3 = \sqrt{\frac{\lambda+2\mu+\alpha}{\rho}}\,, \quad V_4 = \sqrt{\frac{k}{\tau\,(c_v + a)}}\,. \tag{14}$$

Here V_1 is the velocity of propagation of the microrotation wave [14], V_2 and V_3 are the velocities of propagation of the elastic waves, and V_4 is the velocity of the thermoelastic wave (heat wave accompanied by the strain field).

We expand the determinant (13), taking into account all the components ω_{kl}, k and $l = \overline{1, 7}$. We obtain

$$\left((\mu+\alpha)\,g^2 - \rho p_0^2\right)^3 \left((\gamma+\varepsilon)\,g^2 - j\rho p_0^2\right)^2 \left(2b\tau\nu p_0^2\,(\mu g^2 - \rho p_0^2) + (kg^2 - \tau\,(c_v + a)\,p_0^2)\,((\lambda+2\mu)\,g^2 - \rho p_0^2)\right) = 0\,. \tag{15}$$

From Eq. (15) for the velocities of propagation of the discontinuity surfaces we will have

$$P_1 = V_1 = \sqrt{\frac{\gamma+\varepsilon}{j\rho}}\,, \quad P_2 = V_2 = \sqrt{\frac{\mu+\alpha}{\rho}}\,, \quad P_{3,4} = \sqrt{\frac{1}{2}\left(A \pm \sqrt{A^2 - 4B}\,\right)}\,, \tag{16}$$

where

$$A = \frac{\rho k - \tau\,(2\nu b\mu - (\lambda+2\mu)\,(c_v + a))}{\rho\tau c_v}\,, \quad B = \frac{k\,(\lambda+2\mu)}{\rho\tau c_v}\,.$$

In formulas (16), the velocity P_3 belongs to the elastic wave accompanied by the thermal field and the velocity P_4 belongs to the heat wave accompanied by the strain field. As follows from these formulas, the micropolar effects exert no influence on the propagation of thermoelastic waves and lead only to the appearance of new types of waves (in our case, of the wave of microrotations) [14].

TABLE 2. Values of the Velocities of Propagation of Thermoelastic Waves

Material	ρ, kg/m^3	λ, W/(m·deg)	c_v, 10^3 J/(m^3·deg)	Velocities of elastic and thermoelastic waves, m/sec			
				V_3	P_3	V_4	P_4
Silver	10505	418	2454	3762	4321	4054	3593
Lead	11342	34.89	1458	2306	2407	1500	1482
Molybdenum	9010	162	2188	6944	6964	2712	2713

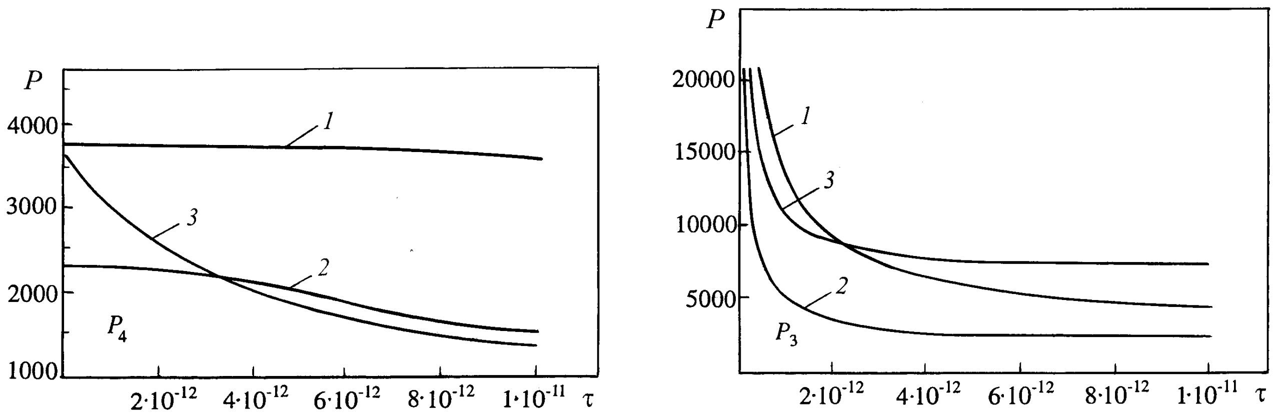

Fig. 1. Velocities of the thermoelastic wave P_4 and P_3 vs. relaxation time of the heat flux τ: 1) silver; 2) lead; 3) molybdenum. P, m/sec; τ, sec.

Let us calculate the velocities V_k and P_k, $k = 3, 4$, of propagation of the elastic and thermoelastic waves in silver, lead, and molybdenum at the temperature $\theta_0 = 293$ K (Table 2) (selection of these metals is attributed to the different character of propagation of the waves).

As follows from Table 2, in silver, the mechanical and thermal fields are connected to a considerable extent, i.e., a temperature change leads to significant strains and conversely. This can be judged from the difference of the velocities P_3 and P_4 from the velocities V_3 and V_4 in value. In molybdenum, $P_3 \approx V_3$ and $P_4 \approx V_4$, i.e., the effects of connectivity of the strain and temperature fields are absent, in practice. The case where the velocity P_3 of the thermoelastic wave virtually coincides with the velocity of the elastic wave is intermediate; the differences manifest themselves in the velocities V_2 and P_2.

We note that in the above calculations the relaxation time of the heat flux τ has been taken to be $1 \cdot 10^{-11}$ sec [13], whereas for the metals the exact values of τ have not been determined and sometimes one takes τ to be $0.5 \cdot 10^{-11}$ sec [15]. Formulas (14) and (16) for V_4 and P_k, $k = 3, 4$, make it possible to investigate the influence of the relaxation time of the heat flux on the velocity of propagation of the thermoelectric waves. Thus, the functions of the velocities of propagation of the thermoelastic waves $V_4(\tau)$ are analogous for all the materials and represent the dependences $V_4 = Kf(\tau)$, where $f(\tau) = \sqrt{1/\tau}$ and K is a coefficient dependent on the mechanical and thermal properties of the material; for $\tau \to 0$ we have $f(\tau) >> K$. Therefore, as the relaxation time of the heat flux decreases the velocity V_3 tends to infinity. With account for the effect of connectivity of the thermal and strain fields the dependence of the velocity of the thermoelastic wave $P_4(\tau)$ can significantly differ from $V_4(\tau)$. Thus, for example, in silver, the velocity P_4 virtually remains constant throughout the interval of change of the time τ from 0 to $1 \cdot 10^{-11}$ sec, whereas in lead and molybdenum the velocities P_4 increase and differ from V_4 only for $\tau \sim 10^{-14}$ sec, as the calculation shows (see Fig. 1). When $\tau \to 0$ the velocity P_4 tends to a finite limit and the velocity $P_3 \to \infty$, which can be interpreted as passage from the generalized theory of heat conduction to a classical theory in which one takes τ to be 0 [16] (see Fig. 1).

By using Eq. (15) we can easily obtain the equations of bicharacteristics which form the characteristic surface and are the components of the group velocity of propagation of the wave. To do this, for example, we express p_0 as follows:

$$p_0 = g\sqrt{\frac{\gamma + \varepsilon}{j\rho}} .$$

The equations of bicharacteristics will take the form [9, 10]

$$\frac{dx_k}{dt} = \frac{dp_0}{dp_k} = \frac{p_k}{g}\sqrt{\frac{\gamma + \varepsilon}{j\rho}} ,$$

or at $t = 1$

$$x_k = \frac{p_k}{g}\sqrt{\frac{\gamma + \varepsilon}{j\rho}} = \cos\alpha_k \sqrt{\frac{\gamma + \varepsilon}{j\rho}} ,$$

where $\cos \alpha_k$ is the direction cosine of the normal to the characteristic surface, $k = 1, 2$ [9, 10]. Then the equation of the surface, formed by the bicharacteristics, upon obvious transformations will take the form

$$x_1^2 + x_2^2 = \frac{\gamma + \varepsilon}{j\rho} .$$

Analogously we can derive the surfaces of bicharacteristics for expressions (15) and (16). The set of all the bicharacteristics will compose the characteristic surface $Z(t, x_1, x_2)$ = const.

Consideration of the three-dimensional case does not introduce fundamental difficulties and can be performed according to the scheme developed above.

REFERENCES

1. I. Nistor, *Bull. Inst. Polytech. Iasi*, Sec. 1, **37**, Nos. 1–4, 88–96 (1991).
2. D. S. Chandrasekaraiah, *Arch. Mech.*, **38**, No. 3, 319–328 (1986).
3. Vl. N. Smirnov, *Inzh.-Fiz. Zh.*, **40**, No. 3, 482–488 (1981).
4. J. Ignaczak, *Therm. Stresses*, **3**, 279–354 (1989).
5. D. E. Grady, *Int. J. Eng. Sci.*, **22**, Nos. 8–10, 1181–1186 (1984).
6. S. Chirita, *J. Therm. Stresses*, **3**, No. 2, 199–221 (1980).
7. V. Novatskii, *Elasticity Theory* [in Russian], Moscow (1975).
8. A. G. Shashkov, V. A. Bubnov, and S. Yu. Yanovskii, *Wave Phenomena of Heat Conduction: System Structural Approach* [in Russian], Minsk (1993).
9. G. I. Petrashen', in: *Collection of Sci. Papers of the Institute of Mathematics of the USSR Academy of Sciences* [in Russian], Leningrad (1984), pp. 3–95.
10. V. I. Smirnov, *A Course in Higher Mathematics* [in Russian], Vol. 4, Pt. 2, Moscow (1981).
11. Modern Crystallography, Vol. 4, *Physical Properties of Crystals* [in Russian], Moscow (1984).
12. I. M. Kikoin (ed.), *Tables of Physical Quantities. Handbook* [in Russian], Moscow (1975).
13. Ya. S. Podstrigach and Yu. M. Kolyano, *Generalized Thermomechanics* [in Russian], Kiev (1976).
14. A. Eringen, *Micropolar Elasticity Theory. Destruction* [Russian translation], Vol. 2, Moscow (1975).
15. H. W. Lord and Y. Shulman, *J. Mech. Phys. Solids*, **5**, No. 15, 299–309 (1967).